Maximum Likelihood Conjoint Measurement in R

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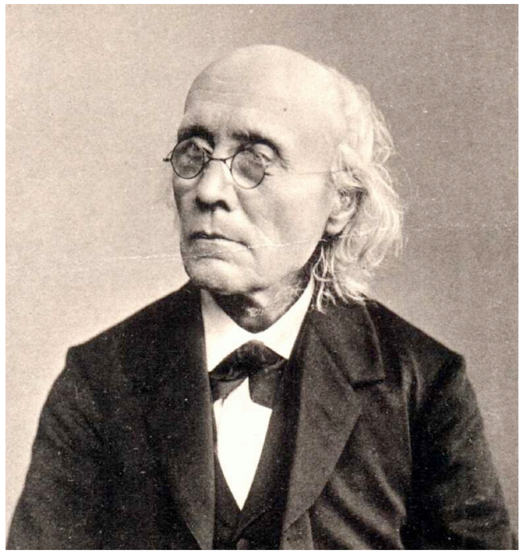
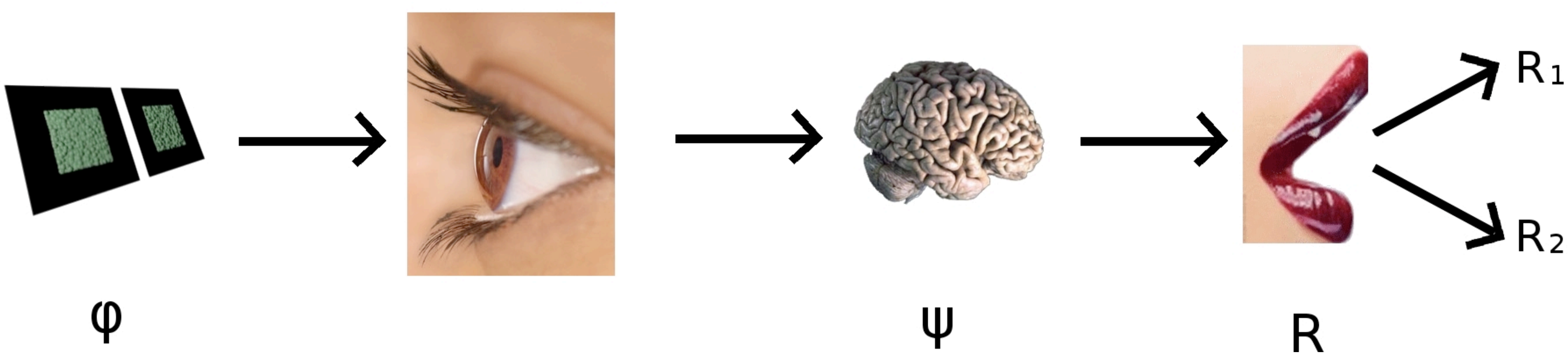
Psychophysics, qu'est-ce que c'est ?



Gustav Fechner (1801 - 1887)

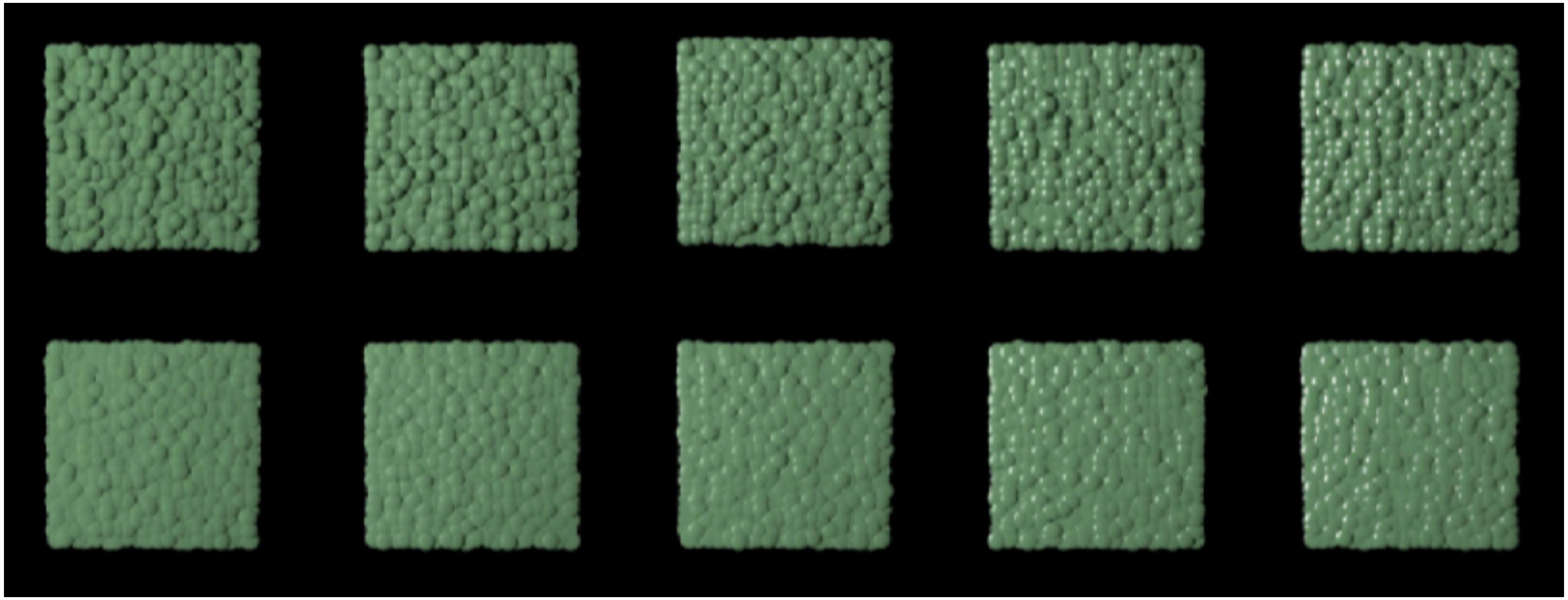
A body of techniques and analytic methods to study the relation between physical stimuli and the organism's (classification) behavior to infer internal states of the organism or their organization.



Conjoint Measurement

Interactions of surface properties: Gloss and 'bumpiness'

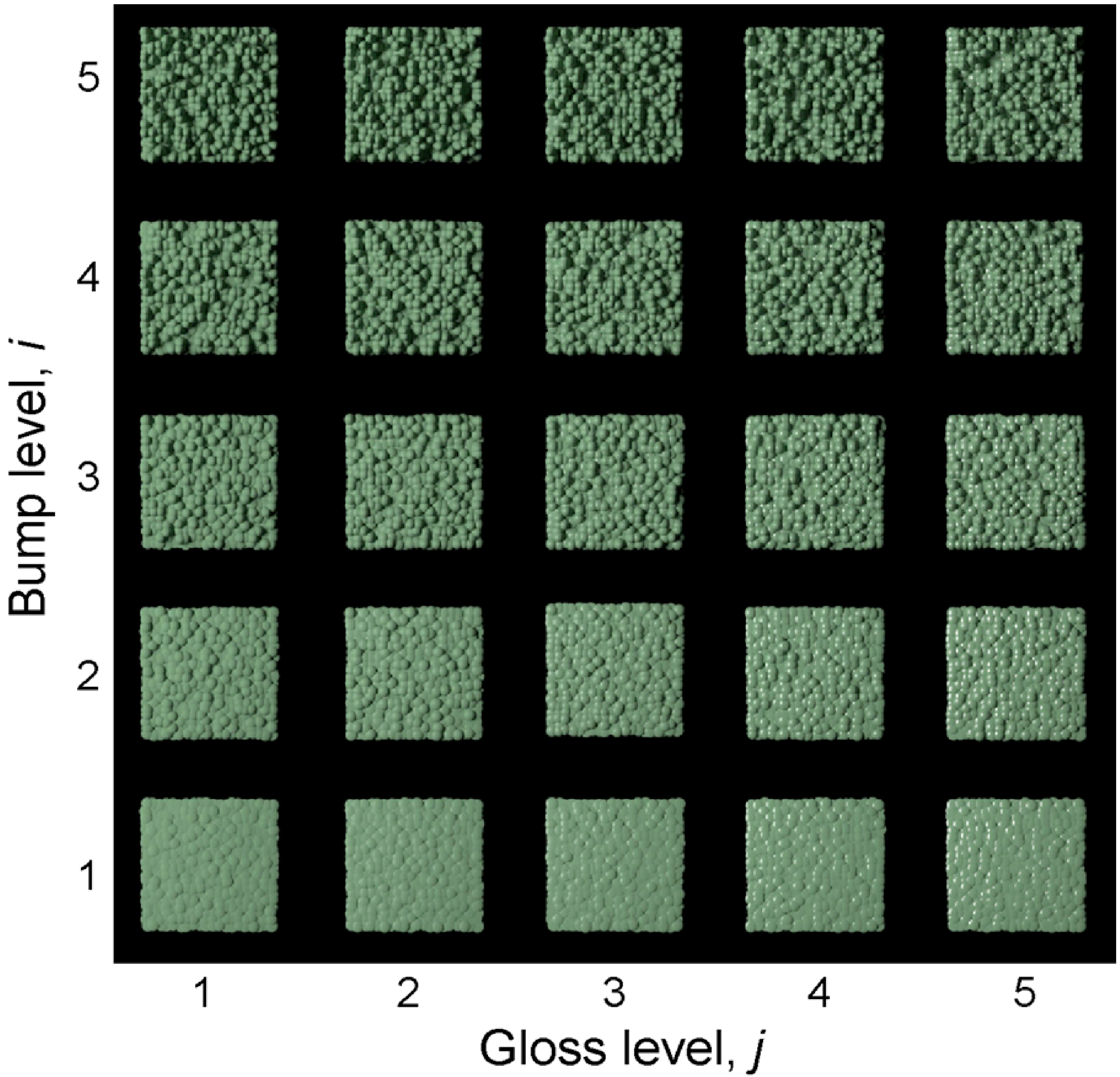
Conjoint Measurement[1] is a psychophysical procedure used to estimate the interaction of perceptual scales for stimuli distributed along $n \geq 2$ physical continua.



[1] *Luce & Tukey (1964) J Math Psych*

Ho, Landy & Maloney (2008) Psych Science

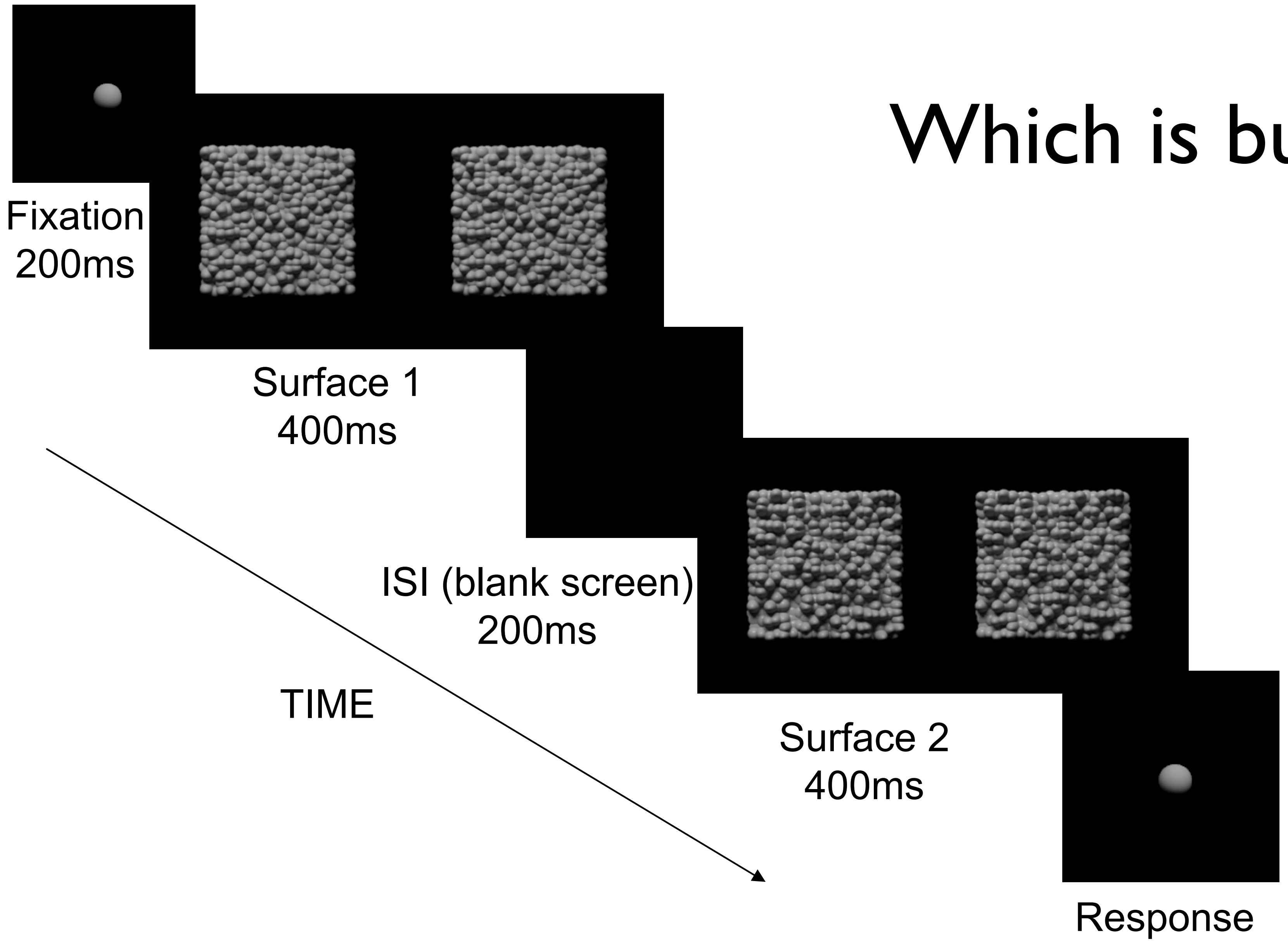
Interactions between Surface Properties



Ho, Landy & Maloney (2008) Psych Science

From a set of p stimuli varying along 2 dimensions, a random pair, (I_{ij}, I_{kl}), is chosen and presented to the observer as in this example.

MLACM: The Task



Which is bumpier (glossier)?

The decision model

$b_1 \; g_1$ $\qquad$ $b_2 \; g_2$

Bumpier?

$$
\begin{aligned}
B_1 &= \psi^b(b_1) + \chi^g(g_1) \\
B_2 &= \psi^b(b_2) + \chi^g(g_2) \\
\Delta &= B_1 - B_2 + \epsilon > 0 \Leftrightarrow \text{“First”}
\end{aligned}
$$

$$\epsilon \sim \mathcal{N}(0, \sigma^2)$$

Ho, Landy & Maloney (2008), Psych Science

Estimation of Scale Values

Ho, Landy & Maloney (2008) used a direct method for estimating the maximum likelihood scale values,

$$L(\mathbf{\Psi}, \sigma) = \prod_{k=1}^{n} \Phi\left(\frac{\delta\left(\mathbf{q}^k\right)}{\sigma}\right)^{1-R_k} \left(1 - \Phi\left(\frac{\delta\left(\mathbf{q}^k\right)}{\sigma}\right)\right)^{R_k}$$

where

$$\Psi = (\psi_2, \cdots, \psi_p, \chi_2, \cdots, \chi_q)$$

$$\delta(q^k) = (\psi^{b_1} + \chi^{g_1}) - (\psi^{b_2} + \chi^{g_2})$$

Φ is the cumulative standard Gaussian (a probit analysis)

R_k is 0/1 if the judgment is left/right image

$\psi_1 = \chi_1 = 0$ and $\sigma = 1$ for identifiability,

leaving $p + q - 2$ parameters to estimate

Estimation of Scale Values

The problem can also be conceptualized as a GLM.

Each level of the stimulus is treated as a covariate in the model matrix, taking on values of 0 or ± 1 in the design matrix,

depending on the presence of the stimulus in a trial and its weight in the decision variable.

	Resp	G1	G2	B1	B2
1	1	3	4	4	3
2	1	3	5	4	2
3	0	1	1	1	4
4	0	2	3	1	2
5	0	1	4	3	4
6	1	1	5	5	2

$$
\begin{array}{cccccccccc}
p_1 & p_2 & p_3 & p_4 & p_5 & q_1 & q_2 & q_3 & q_4 & q_5
\end{array}
$$

$$
\begin{pmatrix}
0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1
\end{pmatrix}
$$

For model identifiability, we drop the first two columns along each dimension, fixing $\psi_1 = \chi_1 = 0$ and $\sigma = 1$.

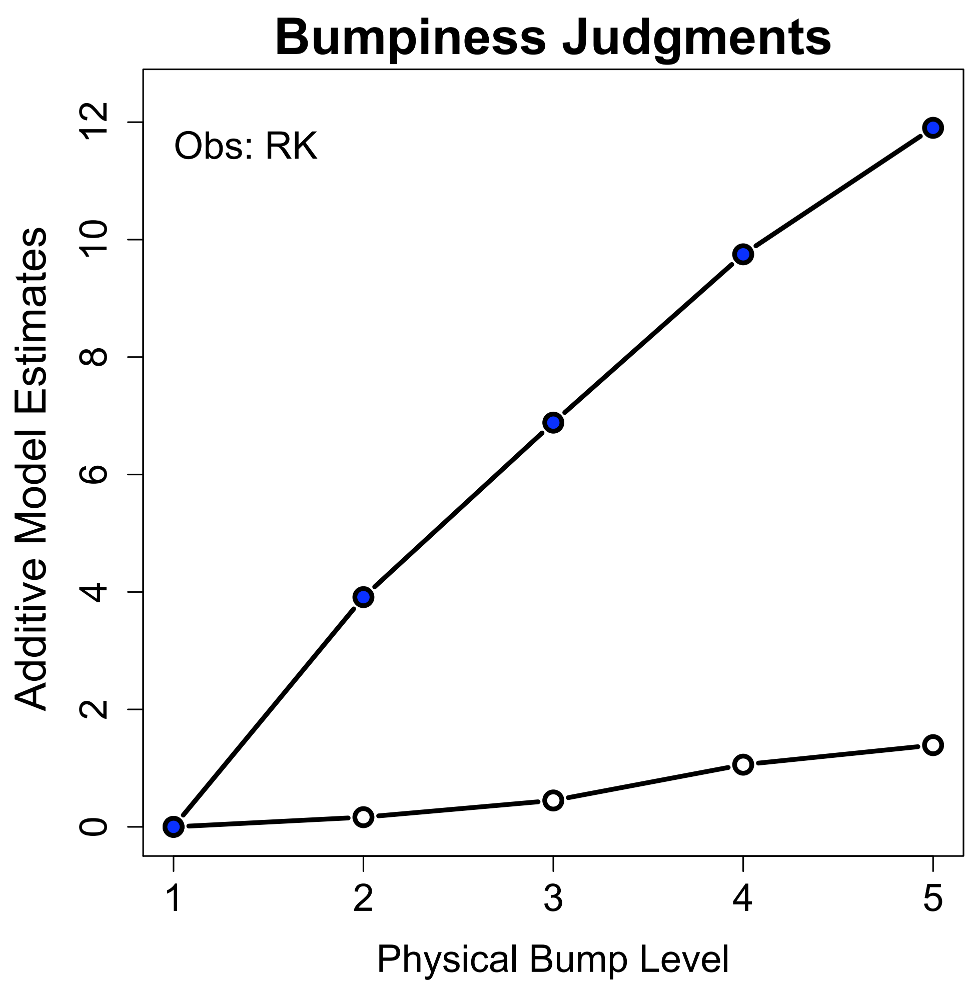
Estimation of Scale Values

```
> head(bg.df)
     Resp G2 G3 G4 G5 B2 B3 B4 B5
[1,]    1  0  1 -1  0  0 -1  1  0
[2,]    1  0  1  0 -1 -1  0  1  0
[3,]    0  0  0  0  0  0  0 -1  0
[4,]    0  1 -1  0  0 -1  0  0  0
[5,]    0  0  0 -1  0  0  1 -1  0
[6,]    1  0  0  0 -1 -1  0  0  1
```

$$\eta\left(\mathrm{E}\left[Y\right]\right) = X\beta$$

```
> glm(Resp ~ . - 1, family = binomial( "probit" ), data = bg.df)
```

The aim of Maximum Likelihood Conjoint Measurement (MLACM) is to estimate scale values, $(\psi_1, \cdots, \psi_p, \chi_1, \cdots, \chi_q)$ whose additive combination best captures the observer's judgments of the perceptual difference between the stimuli in each pair.



The MLACM package[1]

The MLACM package provides a modeling function, mlacm(), that is essentially a wrapper for glm() and will enable estimation of the perceptual scale values, given a data frame with the appropriate structure.

```
mlacm(x,
    model = "add",
    whichdim = NULL,
    lnk = "probit",
    control = glm.control(maxit = 50000, epsilon = 1e-14),
    ...)
```

Default model is "additive", but 2 others may be specified: "independent" (must specify `whichdim`) and "full".

It outputs an S3 object of class 'mlacm' which can be examined further using several method functions:

summary, anova, plot, logLik and AIC

1. Not yet on CRAN

Additive Model

```
> ( bg.add <- mlacm(BumpyGlossy) )

Maximum Likelihood Conjoint Measurement

Model:     Additive
Perceptual Scale:
        G     B
Lev1 0.000 0.000
Lev2 0.132 1.693
Lev3 0.185 2.947
Lev4 0.504 4.281
Lev5 0.630 5.275
```

Independent Model

```
> ( bg.ind <- mlacm(BumpyGlossy, model = "ind", whichdim = 2) )

Maximum Likelihood Conjoint Measurement

Model:     Independence
Perceptual Scale:
    [,1]
B1  0.00
B2  1.66
B3  2.88
B4  4.16
B5  5.11
```

```
> anova(bg.ind, bg.add, test = "Chisq")
Analysis of Deviance Table

Model 1: resp ~ X.B2 + X.B3 + X.B4 + X.B5 - 1
Model 2: resp ~ (X.G2 + X.G3 + X.G4 + X.G5 + X.B2 + X.B3 + X.B4 + X.B5) - 1
   Resid. Df  Resid. Dev  Df  Deviance  P(>|Chi|)
1  971        500.12
2  967        476.48      4   23.64     9.452e-05
```

We can also test a “full” model with 24 parameters!

```
> bg.full <- mlacm(BumpyGlossy, model = "full")

Model:     Full
Perceptual Scale:
      B1       B2       B3       B4       B5
G1  0.000    1.757    2.672    4.094    5.121
G2  0.257   -7.198  -14.141  -15.091  -15.041
G3  0.316   -6.674  -13.647  -14.615  -14.360
G4  0.644   -6.198  -13.275  -13.880  -13.906
G5  0.808  -13.318  -20.783  -21.277  -21.341

> anova(bg.add, bg.full, test = "Chisq")
Analysis of Deviance Table

Model 1: resp ~ (X.G2 + X.G3 + X.G4 + X.G5 + X.B2 + X.B3 + X.B4 + X.B5) - 1
Model 2: resp ~ X.G2 + X.G3 + X.G4 + X.G5 + X.B2 + X.B3 + X.B4 + X.B5 +
    X.G2:X.B2 + X.G3:X.B2 + X.G4:X.B2 + X.G5:X.B2 + X.G2:X.B3 +
    X.G3:X.B3 + X.G4:X.B3 + X.G5:X.B3 + X.G2:X.B4 + X.G3:X.B4 +
    X.G4:X.B4 + X.G5:X.B4 + X.G2:X.B5 + X.G3:X.B5 + X.G4:X.B5 +
    X.G5:X.B5 - 1
  Resid. Df Resid. Dev  Df Deviance P(>|Chi|)
1       967     476.48
2       951     451.66  16    24.82      0.07
```

Testing of Bias of MLACM model

Simulated Observer with response to a stimulus defined as weighted combination of responses to 2 stimulus dimensions (A, B):

$$R = S_A^{p_1} + w S_B^{p_2}$$

Decision Variable:

$$\Delta = R_1 - R_2 + \epsilon, \qquad \epsilon \sim \mathcal{N}(O, \sigma^2)$$

Decision Rule:

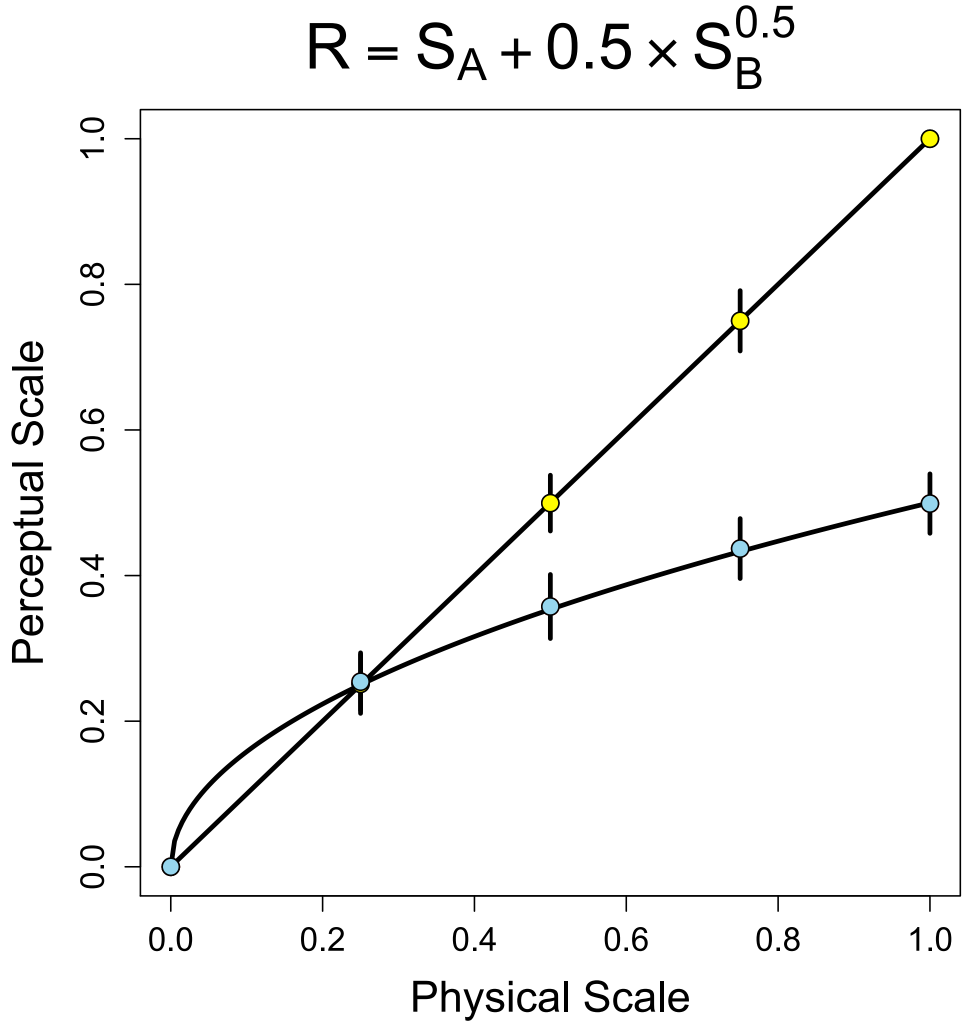
$$\begin{aligned} \Delta \quad > \quad & 0, \quad \text{choose ``First''} \\ & \text{else choose ``Second''} \end{aligned}$$

$$\begin{aligned} S &= (0, 0.25, 0.5, 0.75, 1) \\ p &\in \{0.25, 0.5, 1, 2, 4\} \\ w &\in \{0.1, 0.5, 0.8\} \end{aligned}$$

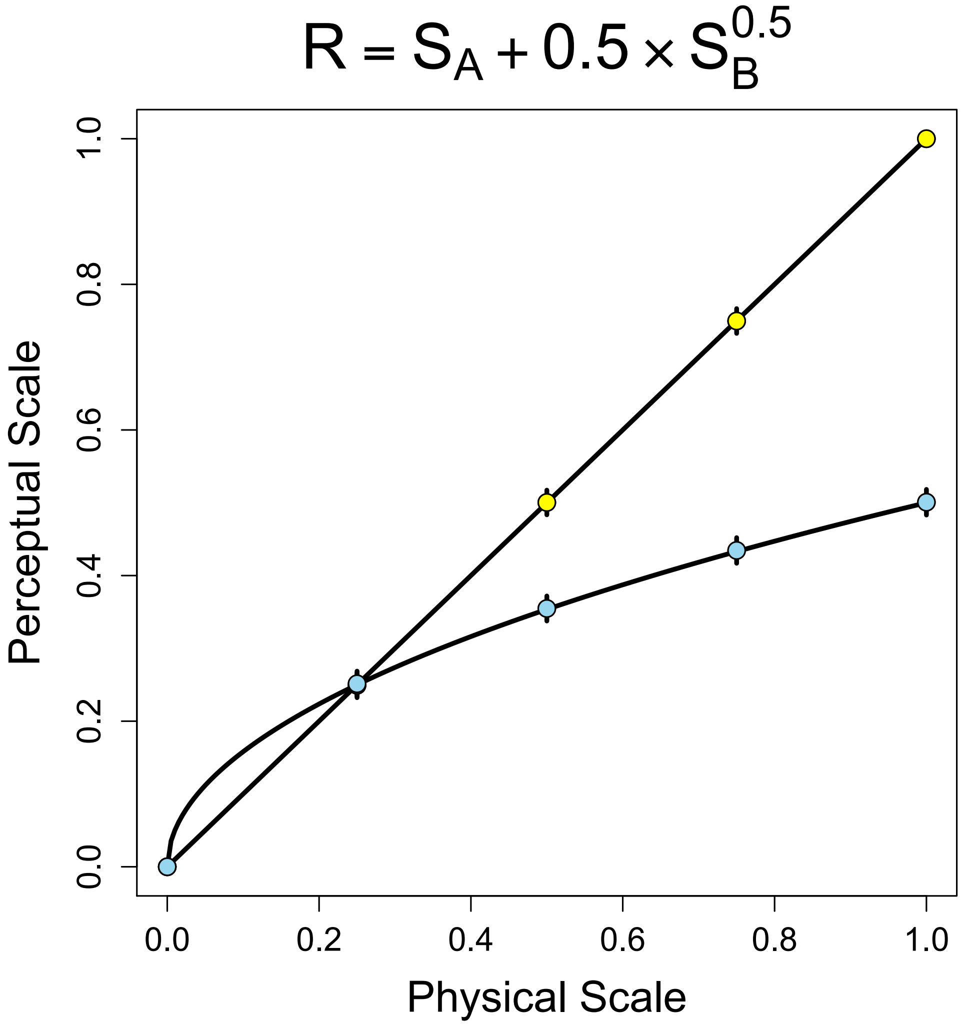
Bias and Number of Trials

Mean and SD of 1000 experiments with predicted curves

1 replication, 300 Trials



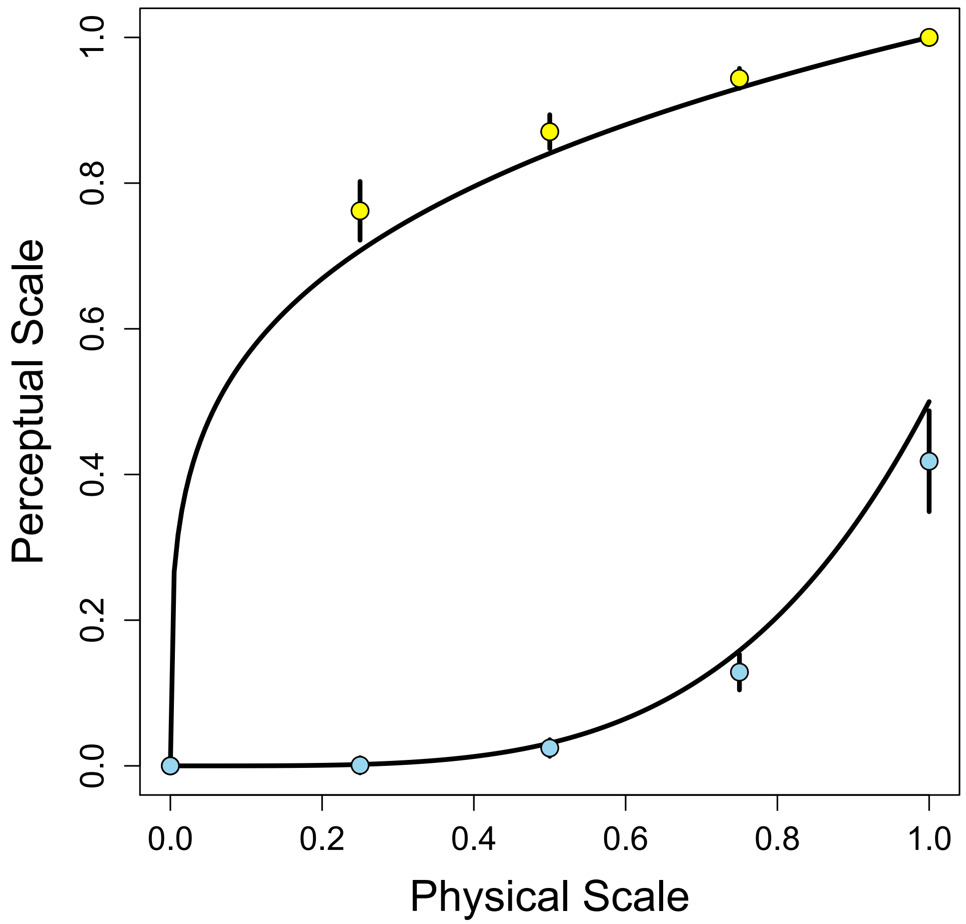
5 replications, 1500 Trials



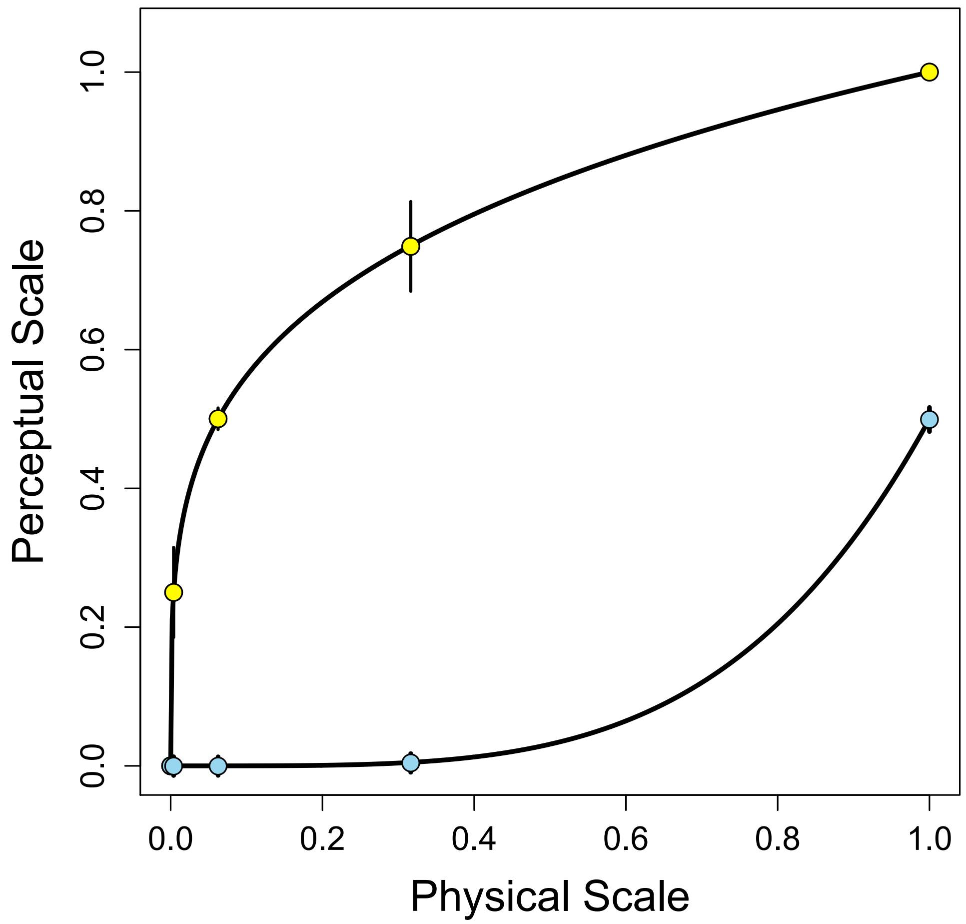
Bias and Bias Correction

Bias arises when one curve is too flat, ...

$$R = S_A^{0.25} + 0.5 \times S_B^4$$



$$R = S_A^{25} + 0.5 \times S_B^4$$



but can be corrected by respacing the stimuli along the physical scale.

Additional tests demonstrate that:

i. the variability of the estimates depends on the number of trials and not the number of physical scale values.

ii. the estimates are relatively robust to distributional assumptions

iii. the estimates are relatively robust to inhomogeneity of the variance

Future Directions

i. Add a method to generate bootstrap standard errors of estimated scale values

ii. Add diagnostic tests of the fits based on the residuals

iii. Add a formula method to fit parametric models to the data

iv. Finish documentation and submit the package to CRAN ;^)

v.   ...

Summary

i. Conjoint Measurement is a procedure to examine how physical dimensions interact in perceptual judgments.

ii. We presented functions for fitting data from a Conjoint Measurement experiment using maximum likelihood methods (via glm) and a package, MLACM (soon to be released).

iii. Simulations with a model observer show that bias in the estimations depends on the rate of change of the underlying responses and the number of trials, but that when these are adjusted the estimates are quite robust with respect to distributional and variance assumptions.