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# Estimating Markovian Switching Regression Models in R
# An application to model energy price in Spain

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# Outline

# 1. Introduction

The model we consider is of the **MARKOVIAN SWITCHING (MS)** type, originally defined by **Hamilton** (1989).

- MSVAR library - Krolszing (1998) (not available free acces: OX)
- MSVARlib - Bellone (2005) (Less user friendly)
- MSRegression - Perlin (2007) (Libraries in Matlab)

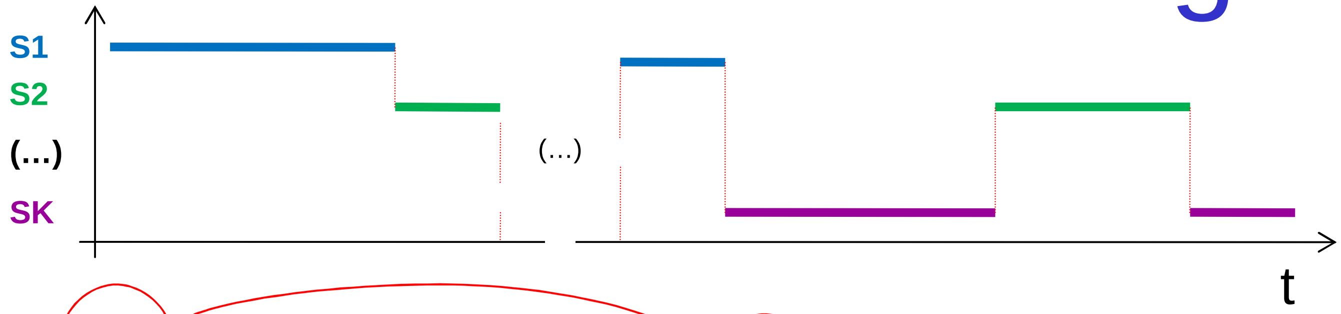
# 1. Objectives

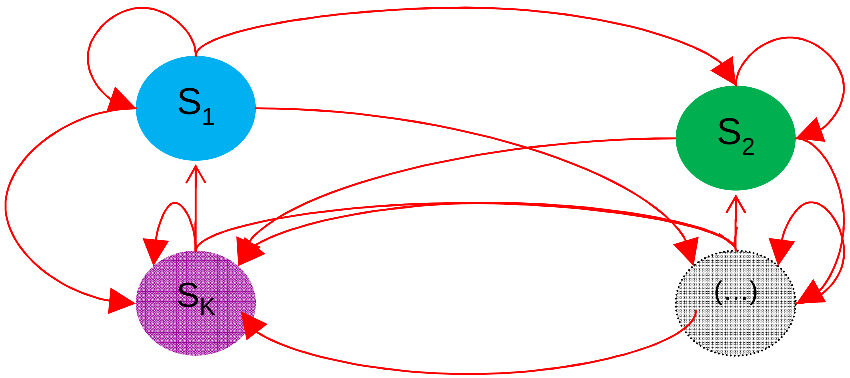
1. Built a set of R functions to explain time series according to a Markovian Switching Regression model.

2. Resolution of the problems during the estimation of the Markovian Switching models.

3. Application of Markovian Switching models in energy price in Spain according to the demand, raw material prices and financial indicators.

# 2. Markovian Switching



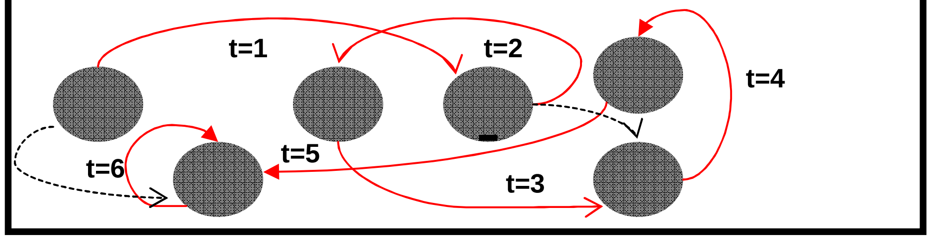




$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix}$$

## MARKOVIAN

$$P(S_t=i \mid S_{t-1}=j, S_{t-2}=k,\ldots,S_1=k)= P(S_t=i \mid S_{t-1}=j)$$

## SWITCHING

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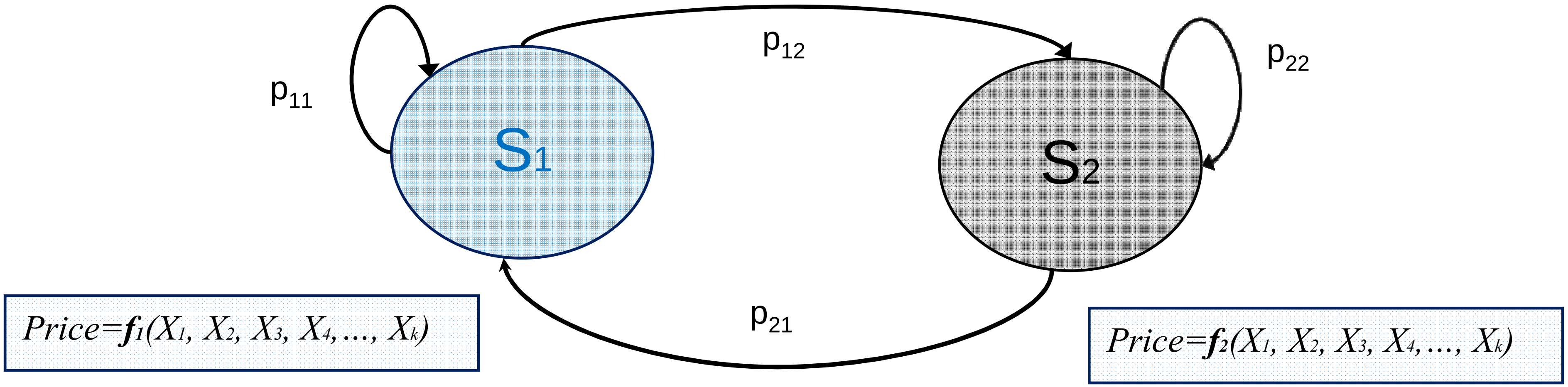
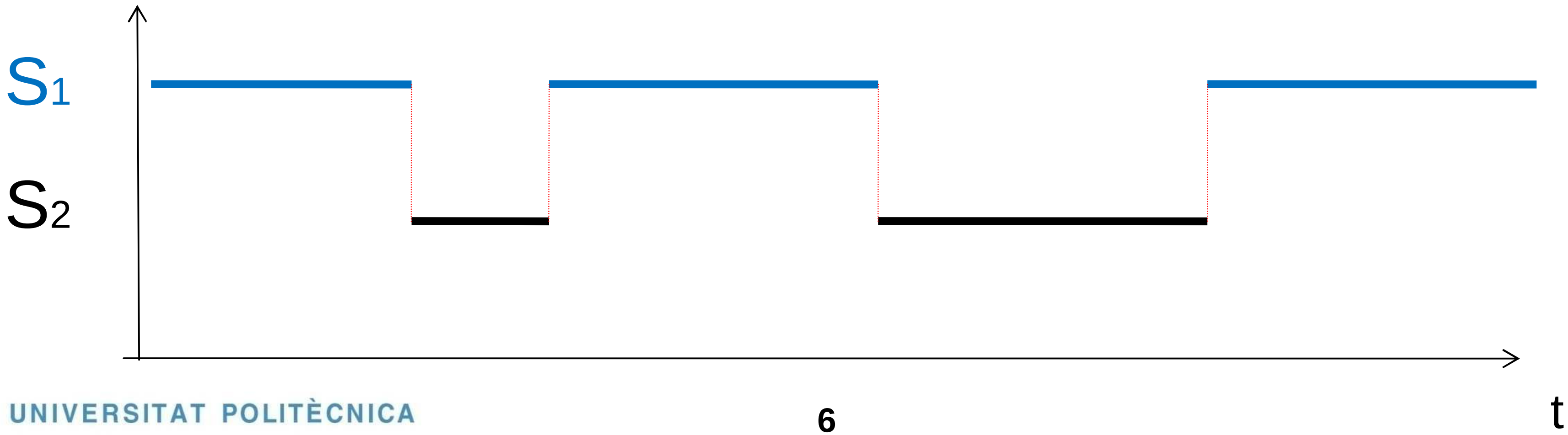
# 2. Markovian Switching



# 2. Markovian Switching

**S : Number of states**

$$P_{i,t} = \begin{cases} \beta_{1,t}^{(1)} X_{1,t} + \cdots + \beta_{k,t}^{(1)} X_{k,t} + \beta_{k+1,t} X_{k+1,t} + \cdots + \beta_{j,t} X_{j,t} + \varepsilon_{t,1} & S_t = 1 \\ \beta_{2,t}^{(2)} X_{1,t} + \cdots + \beta_{k,t}^{(2)} X_{k,t} + \beta_{k+1,t} X_{k+1,t} + \cdots + \beta_{j,t} X_{j,t} + \varepsilon_{t,2} & S_t = 2 \end{cases}$$

VARIABLES WITH SWITCHING EFFECT | VARIABLES WITHOUT SWITCHING EFFECT

**Parameters of the model to estimate:**

Deviations of the states: $(\sigma^{(1)}; \sigma^{(2)})$

Coefficients of the regression with Switching effect : $(\beta_{1,\ldots,k}^{(1)}; \beta_{1,\ldots,k}^{(2)})$

Coefficients of the regression without Switching effect : $(\beta_{k+1,\ldots,}\beta_{t})$

Transition probabilities: $(p_{11,}p_{22})$

**INPUT** (user)

- Dependent variable ($P_t$)
- Independent variables (X's)
- Number of states (S)
- Switching Indicator

Perlin (2005)

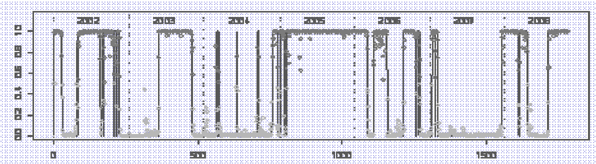
→ **MODEL FITTING**

↓ **LM**

**OUTPUT**

**Graphic**

- Evolution of State assignation with probabilities



**Numeric**

Parameters Estimation ($\theta$)

For each time instant…

- State Assignation
- Probability assignment in each state

Model parameters:

$$\Theta = (\beta, \sigma, \Pi)$$

Model Likelihood:

$$L(\Theta; y_{1:T}, X_{1:T}) = f(y_{1:T} \mid X_{1:T}, \beta, \sigma) =$$

$$\sum_{t=1}^{T} \sum_{S} f(y_t \mid S_t, X_t, \beta, \sigma) P(S_t \mid \Pi)$$

The state S is a non-observable latent variable ➔ Likelihood = marginal of the conjoint density for $y$ and S

In this case, the functional dependence between $y$ and $X$ corresponds to a linear model (OLS)

$$y \mid X, \beta, \sigma \sim N(X\beta, \sigma^2 I)$$

Conditioning on the state S means a different set of parameters for each state.

$$y \mid S, X, \beta, \sigma \sim N(X\beta^{(S)}, {\sigma^{(S)}}^2 I)$$

Other set-up can be considered:

- Extending predictors ➔Autoregressive models
- Modifying response distribution ➔Generalized LM
- More complex functional dependence➔ Non-linear models

# 3. Application – Energy price

Electricity markets are characterized by:

- inelasticity of the demand
- impossibility of storage
- Seasonality character: fluctuations of demand due to weather conditions and human habits

In the last decade, the issue of modeling and forecasting prices had been the key question to:

- determine the causes of price behavior
- Macroeconomic significance of the prices of raw materials. Spain is an importer country

# 3. Application – Energy price

The objective of the application is to identify the influence on the energy price of:

- the demand
- the price of the raw materials
- financial information of the markets

during different states of its evolution.

Development of an R Code to estimate MSM

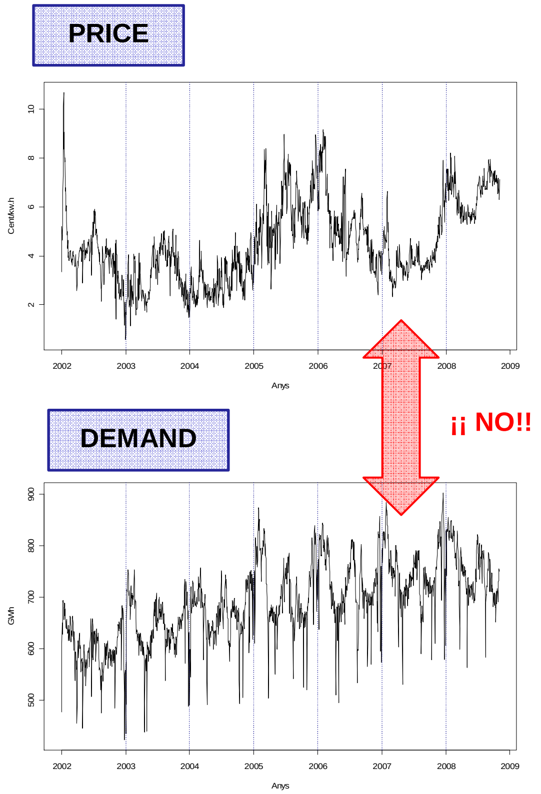
# 3. Data

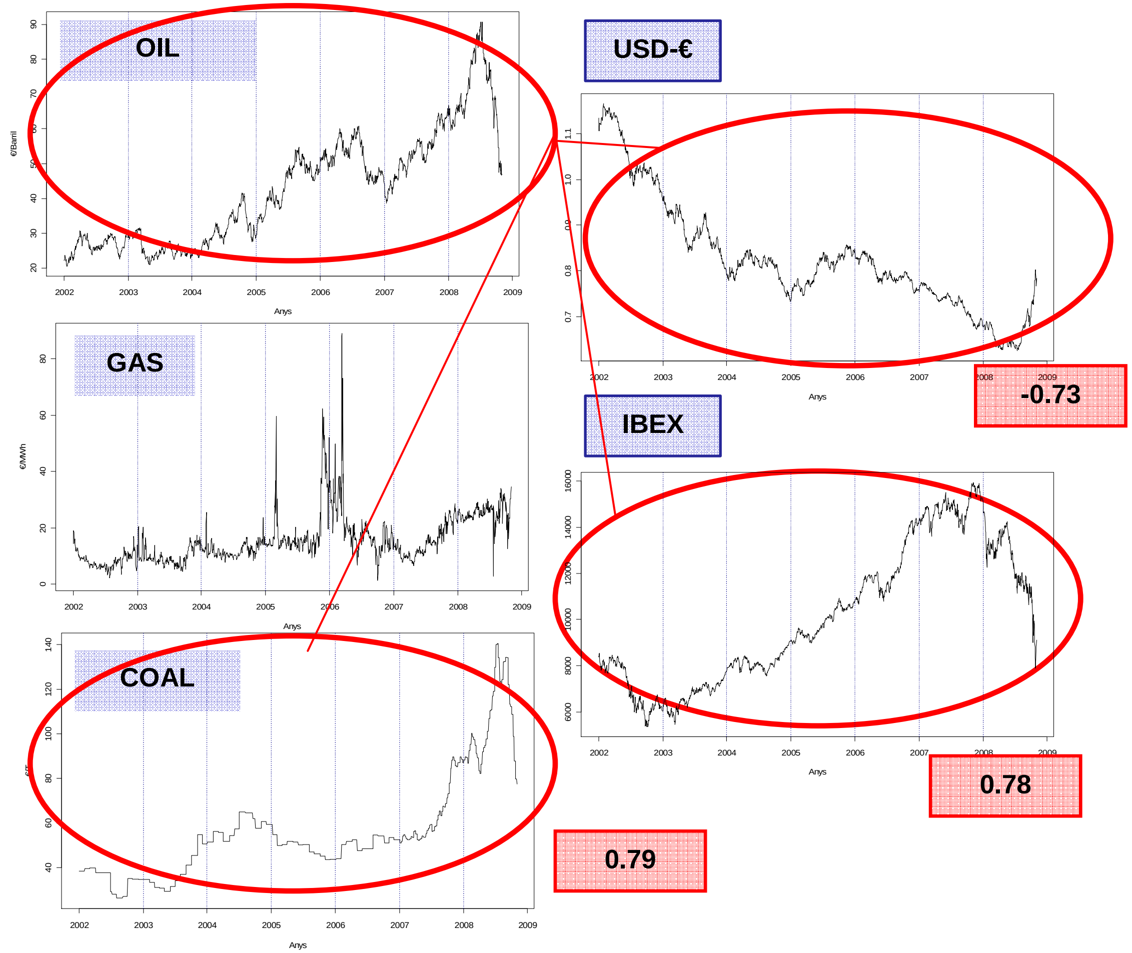
**Data from January 1, 2002 to October 31, 2008**
**(daily data– working days: Monday to Friday)**

| ENERGY | RAW MATERIALS | FINANCIAL |
|---|---|---|
| • Average price of energy (Cent/Kw.h)<br>• Daily demand of energy (GWh) | • Oil Price (€/barril)<br>• Gas Price (€/MW.h)<br>• Coal Price (€/T)<br>• Price of $CO_2$ Allowances (€/T) | • Exchange Rate between Dolar - Euro (USD-Euro)<br>• Ibex 35 Index |

Bierbrauer, Truck and Weron (2006)
Amano and Norden (1998); Zachmann (2007)

# 3. Data





```
>ts.plot(data)
```

# 4. Estimation Procedure

MAXIMUM LIKELIHOOD ESTIMATION OF THE PARAMETERS

$$\theta = \begin{Bmatrix} \beta_0, \beta_{Demand}, \beta_{Oil}, \beta_{Gas}, \beta_{Coal}, \beta_{USDE}, \beta_{Ibex} \\ \sigma_S^{(1)}, \sigma_S^{(2)}, p_{11}, p_{22} \end{Bmatrix}$$

Number of parameters: 18 ➡ Convergence not assured!!

1. How to determine starting values:

   Considering “No switching”

   - Same model under both regimes
   - Probability of change equal to 0.5

   ➔ Estimate linear model (OLS) with all the observations

# 4. Results

$$P_{i,t} = \beta_0 + \beta_{Demand,t} + \beta_{Oil,t} + \beta_{Gas,t} + \beta_{Coal,t} + \beta_{USD/E,t} + \beta_{Ibex35,t} + \varepsilon_t$$

| β | **Estim.** | **Std.Error** |
|---|---|---|
| βConst. | -9.046 | 0.54006*** |
| βDemand | -0.0090 | 0.0004*** |
| βOil | 0.0832 | 0.0037*** |
| βGas | 0.0420 | 0.0039*** |
| βCoal | -0.00819 | 0.00201 |
| βUSD/E | 6.059 | 0.3646*** |
| βIbex35 | -0.0001 | 0.00001*** |

$R^2$=0.57 σ=1.06

# 4. Estimation Procedure

## MAXIMUM LIKELIHOOD ESTIMATION OF THE PARAMETERS

**EM Alg.**

2. Non-linear optimization (Newton-Raphson)

   Evaluation of the likelihood of the model

   ➔ Function *optim* to find MLE

3. Expectation step for $S_t$

   Calculate the Expectation of $S_t$ under the current estimates of the parameters

   ➔ Assign each observation to one of the states

4. Maximization step for parameters

   Conditioning on the values for $S_t$, obtain new estimates

   ➔ Estimate linear model (OLS) for each state

   ➔ go to step 2 until convergence

# 4. Numerical Results

```
State=1
-------
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -9.575e+00  6.425e-01 -14.901  < 2e-16 ***
Demanda      1.115e-02  5.601e-04  19.898  < 2e-16 ***
Petroli      8.161e-02  4.924e-03  16.575  < 2e-16 ***
Gas          2.192e-02  4.212e-03   5.204 2.44e-07 ***
Carbo        1.406e-03  2.097e-03   0.670    0.503
EurDol       6.219e+00  4.189e-01  14.844  < 2e-16 ***
Ibex35      -1.850e-04  2.337e-05  -7.918 7.30e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8421 on 873 degrees of freedom
Multiple R-squared: 0.6648,     Adjusted R-squared: 0.6625

Residuals:
     Min       1Q   Median       3Q      Max
-1.90943 -0.35887 -0.01232  0.37846  1.53960

State=2
-------
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.361e+00  3.751e-01 -19.623  < 2e-16 ***
Demanda      5.301e-03  3.155e-04  16.799  < 2e-16 ***
Petroli      5.371e-02  2.539e-03  21.151  < 2e-16 ***
Gas          2.338e-02  3.148e-03   7.428 2.57e-13 ***
Carbo        1.350e-02  1.613e-03   8.371  < 2e-16 ***
EurDol       5.106e+00  2.566e-01  19.902  < 2e-16 ***
Ibex35      -2.486e-05  9.812e-06  -2.534   0.0115 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5229 on 897 degrees of freedom
Multiple R-squared: 0.8105,     Adjusted R-squared: 0.8093
```

Model under State 1

Model under State 2

Transition Matrix

```
Transition Matrix
-----------------
          1      2
   1 0.9715 0.0487
   2 0.0284 0.9512

Likelihood:  1875.747

Pooled Residuals:

Residual standard error: 0.69659 on 1770 degrees of freedom
Multiple R-squared: 0.816691
```

# 4. Numerical Results

$$P_{i,t} = \begin{cases} \beta_0^{(1)} + \beta_{Demand,t}^{(1)} + \beta_{Oil,t}^{(1)} + \beta_{Gas,t}^{(1)} + \beta_{Coal,t}^{(1)} + \beta_{USD/E,t}^{(1)} + \beta_{Ibex35,t}^{(1)} + \varepsilon_t^{(1)} & S_t = 1 \\ \beta_0^{(2)} + \beta_{Demand,t}^{(2)} + \beta_{Oil,t}^{(2)} + \beta_{Gas,t}^{(2)} + \beta_{Coal,t}^{(2)} + \beta_{USD/E,t}^{(2)} + \beta_{Ibex35,t}^{(2)} + \varepsilon_t^{(1)} & S_t = 2 \end{cases}$$

| β | Estim. | Std.Error |
|---|---|---|
| βConst. | -9.575 | 0.6421*** |
| βDemand | 0.0115 | 0.0005*** |
| βOil | 0.08161 | 0.0049*** |
| βGas | 0.02192 | 0.0049*** |
| βCoal | 0.00014 | 0.0021 |
| βUSD/E | 6.219 | 0.418*** |
| βIbex35 | -0.0008 | 0.00001*** |

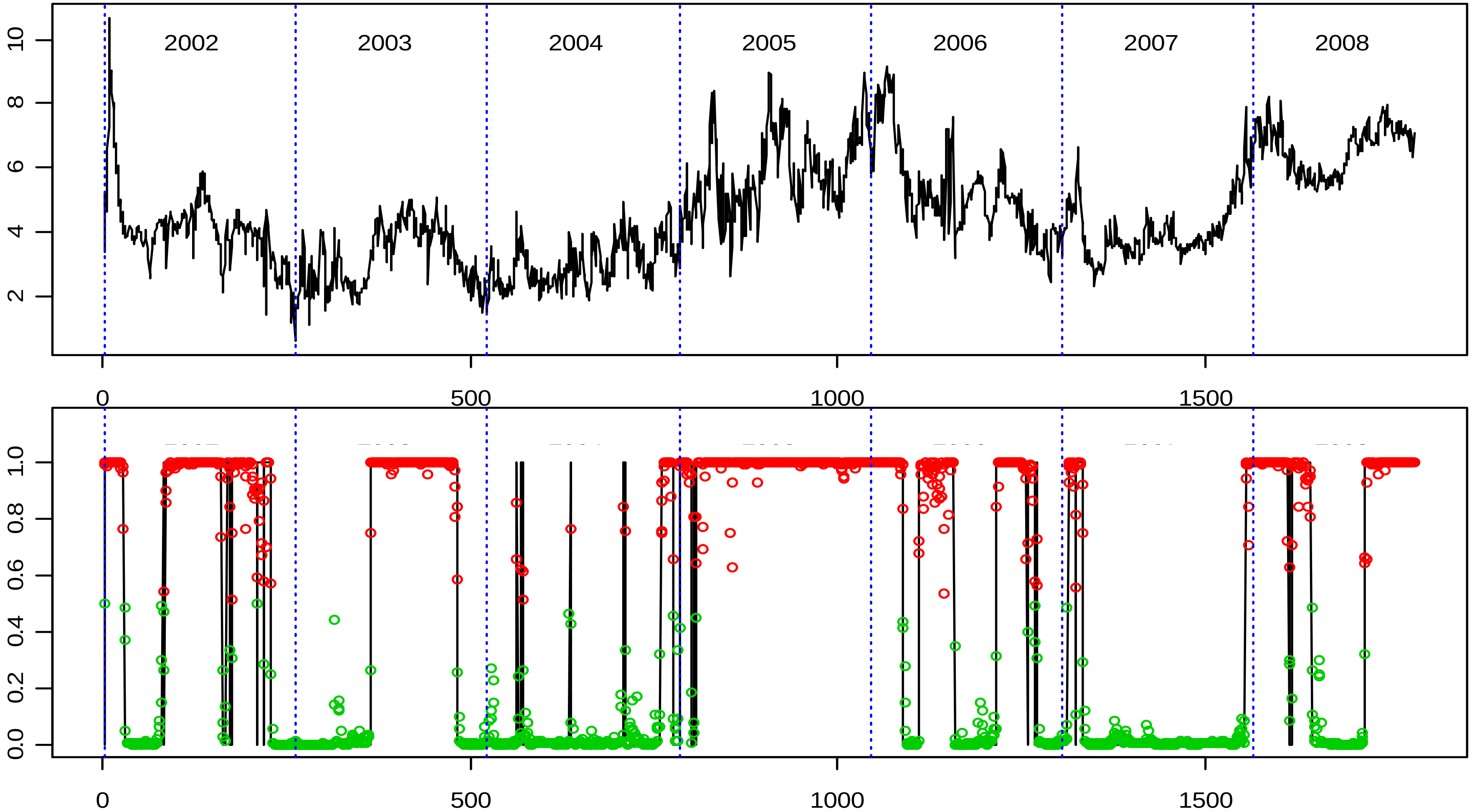
σ=0.842 | $R^2$=0.66

| β | Estim. | Std.Error |
|---|---|---|
| βConst. | -7.3610 | 0.37515*** |
| βDemand | 0.005301 | 0.0003*** |
| βOil | 0.05371 | 0.00253*** |
| βGas | 0.0238 | 0.0031*** |
| βCoal | -0.01352 | 0.00162*** |
| βUSD/E | 5.1062 | 0.256*** |
| βIbex35 | -0.00002 | 0.00009*** |

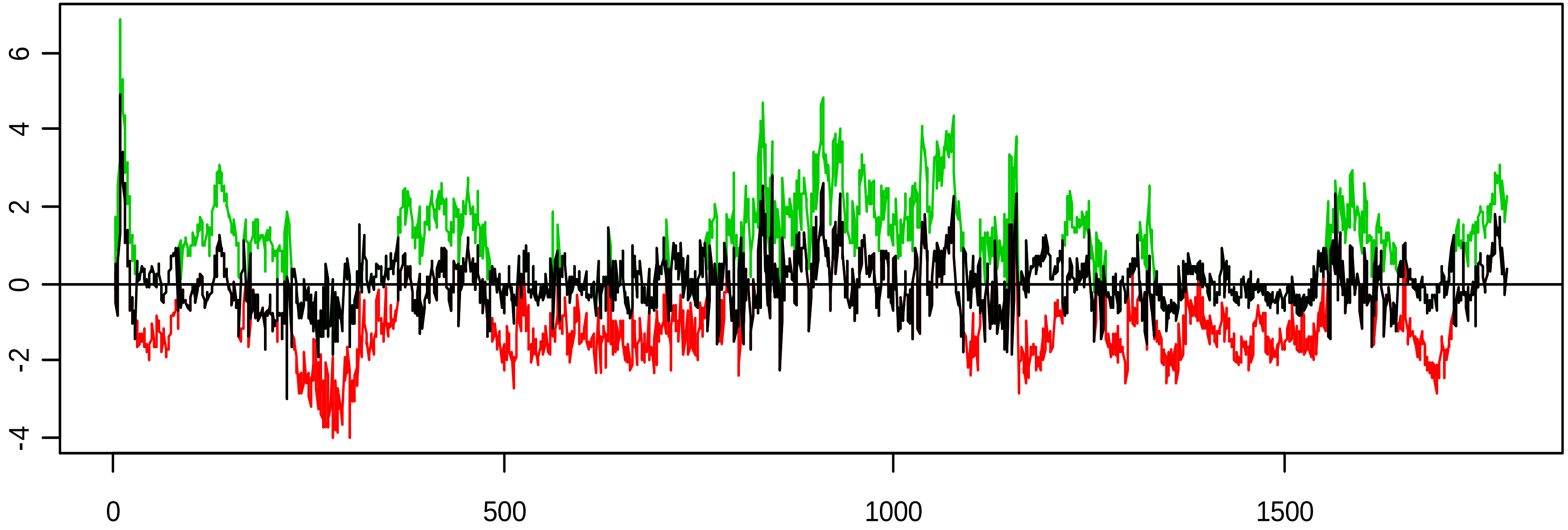
σ=0.52 | $R^2$=0.81

$$P = \begin{bmatrix} 0.98 & 0.05 \\ 0.02 & 0.95 \end{bmatrix}$$

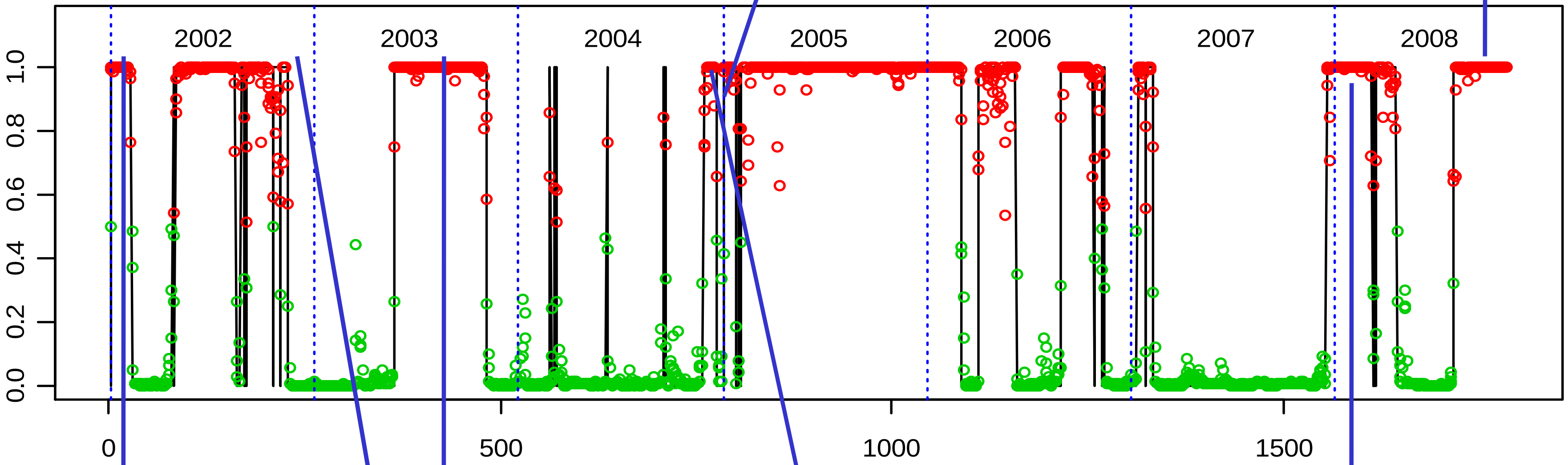
# 4. Graphical Results



# 4. Graphical Results



# 4. Results


2002
2003
2004
2005
2006
2007
2008
0.0
0.2
0.4
0.6
0.8
1.0
0
500
1000
1500
Tsunami
Economic Recession
Atypical Situation. Weather Cause
Gulf War. Iraq War.
Rising Oil Prices
“The Corralito”
Yukos

# 5. Conclusions

1. **Detection of a model that is not constant over the time**
2. **Implementation of a estimation methodology for Markovian Switching models (2 states) using R**
3. **Relationship between states changes of price and USD-Euro and Ibex35**
4. **State changes in energy price caused by different movements in raw materials price (oil and gas).**

# 5. Future Lines

**Work on the routines to be a unique function that allows to the user:**

1. **Reaction time on the market:** Introduce some lags information of explanatory variables.
2. **Autoregressive terms:** Introduce previous prices as explanatory variables.
3. **Orthogonal Model:** Consider the components of PCA and FA as explanatory variables.
4. **Flexibility of the model:** Check the need of switching effect. Increase the number of states.

# References

Bierbrauer, M., Trück, S., Weron, R., 2003. "*Modeling Electricity Prices with Regime Switching Models*", Phisica A 336, 39-48.

Goldfeld, S., Quantd, R., 2005. "*A Markov model for switching Regression*",Journal of Econometrics 135, 349-376.

Hamilton, J., 2005. "*Regime Swtiching Models*", La Jolla CA 92093-0508.

Perlin, M., 2007. "*Estimation, Simulation and Forecasting of a Markov Switching Regression*", (General case in Matlab).

Zachmann, G., 2006. "*A Markov Switching model of the merit order to compare British and German Price formation*", Discussion paper. German Institute for Economics Research

# Thank you

# use R's!